A dynamic model of atoms:
structure, internal interactions and photon emissions of hydrogen

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The standard solution of the Schrödinger equation for the hydrogen atom is analyzed. Comparing with the recently established internal properties of electrons it is found, that these solutions cannot be seen as physically valid states of the electron wave. The paper therefore proposes a new model of hydrogen based on internal properties of electrons. The ground state of the hydrogen system (T=0) is an inertial aggregation within the atomic shell, the calculation yields an atomic radius of 0.330 nm. Electron proton interaction within the atom are treated with a causal and deterministic model, the resonance frequency of the hydrogen system $\nu_0 = 6.57 \times 10^{15} Hz$ is referred to dynamic charge of its nucleus, resonance levels are a result of boundary conditions for radial electron waves and photon interactions due to nuclear oscillations. Spectral emissions of excited atoms can be referred to a decay of the state of motion of the coupled electron–proton system. The framework developed is essentially deterministic, microphysical processes analyzed are referred to material characteristics of particles involved. Statistical effects are referred to interactions with the atomic environment, the results derived are compatible with the second and third principle of thermodynamics.

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I. INTRODUCTION

In recent publications [1–4] we demonstrated, that fundamental relations of quantum theory have to be understood as a consequence of intrinsic particle structures. The discovery of intrinsic potentials, inherent to wave properties of moving particles described by the modified de Broglie relations, allowed to deduce the fundamental relations of quantum theory as well as electrodynamics, and it was shown, that these intrinsic properties do possess physical significance.

The Schrödinger equation [5] was identified as a modification of the wave equation including external potentials, and it was demonstrated that in its traditional formulation it does not account for intrinsic potentials, which makes its fundamental results arbitrary. Heisenberg's uncertainty relations were deduced by estimating the effect of this arbitrariness.

We could equally show, that the interaction of electrons and photons accounts for the observed quantization, although quantization must be understood as a result of transfer processes at every single point of a given region of interaction. The analysis of electrostatic interactions finally allowed the conclusion that electrostatic fields are determined by photon interactions.

In this paper we analyze the traditional model of hydrogen atoms derived from solutions of Schrödinger's equation including a central electrostatic potential, and develop the consequences for material waves. Based on the results of this analysis the experimental constraints are determined, which must be observed in the construc-

tion of an alternative model compatible with the intrinsic wave features of electrons. Finally, by formalizing the process of photon emission, we derive a deterministic theory of electron states within the hydrogen atom, which includes the electromagnetic fields of emission.

II. CURRENT STANDARD

The atomic structure, as well as the, then puzzling, experimental results in atomic physics were the main motivation for constructing the theoretical framework of quantum theory. It was, basically, the only way out of the profound dilemma of physics after the first attempts by Bohr and Sommerfeld – today known as the "old quantum theory" [6,7] – failed, to explain measurements by a model consistent with classical mechanics enhanced by Einstein's concept of photons [8].

A. Solutions of Schrödinger's equation

Limiting calculations of hydrogen ground–states to the Hamiltonian of electron motion in the field of a central charge +e, not accounting for spin–properties and electron–proton interactions as well as neglecting applied electrostatic or magnetic fields, the Schrödinger equation reads [5]:

$$\left(-\frac{\hbar^2}{2m}\triangle - \frac{e^2}{r} \right) \psi(\vec{r}) = E\psi(\vec{r}) \tag{1}$$

The standard procedure separates the radial and lateral components of $\psi(\vec{r})$ (see, for example [9]).

$$\psi(\vec{r}) = \frac{y_l(r)}{r} Y_l^m(\vartheta, \varphi) \qquad (2)$$

Separating the Laplace operator in spherical coordinates:

$$\triangle = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{r^2\sin\vartheta}\left(\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta} + \frac{1}{\sin\vartheta}\frac{\partial^2}{\partial\varphi^2}\right)$$

using the eigenvalue equation for the spherical harmonics $Y_l^m(\vartheta, \varphi)$:

$$\frac{1}{\sin\vartheta}\left(\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta} + \frac{1}{\sin\vartheta}\frac{\partial^2}{\partial\varphi^2}\right)Y_l^m = l(l+1)Y_l^m \quad (3)$$

the Schrödinger equation leads to the radial equation:

$$y_l'' + \left(\frac{2mE}{\hbar^2} + \frac{2m}{\hbar^2}\frac{e^2}{r} - \frac{l(l+1)}{r^2}\right)y_l = 0 \qquad (4)$$

A substitution of variables $x = 2r\sqrt{-\frac{2mE}{\hbar^2}}$ yields for the regular solution of $y(x)$ the expression:

$$y_l(x) = x^{l+1}e^{-x/2}\sum_{p=0}^{n'}\frac{n'!(2l+1)!}{(n'-p)!(2l+1+p)!p!} \qquad (5)$$

The discrete values of n' and l, the *quantum numbers* derive from the requirement of regularity for the Laguerre polynomials [9], the quantum numbers for the ground state of a hydrogen atom are consequently:

$$n = n' + l + 1 \qquad n, n', l \in N \qquad (6)$$

The discrete energy values are a consequence of the same condition, and the energy levels are therefore:

$$n = \frac{e^2}{\hbar}\sqrt{-\frac{m}{2E_n}} \quad \Rightarrow \quad E_n = -\left(\frac{e^2}{\hbar}\right)^2\frac{m}{2n^2} \qquad (7)$$

The wave functions of the two lowest states n = 1,2 are described by:

$$\psi_{100}(\vec{r}) = \frac{2}{a^{3/2}\sqrt{4\pi}}e^{-r/a} \qquad a = \frac{\hbar^2}{me^2}$$

$$\psi_{200}(\vec{r}) = \frac{1}{\sqrt{8\pi}}\left(1 - \frac{r}{2a}\right)e^{-r/2a} \qquad (8)$$

$$\psi_{210}(\vec{r}) = \sqrt{\frac{1}{4\pi}}\frac{r}{2a}e^{-r/2a}\cos\vartheta$$

B. Dynamics of the electron

As recently demonstrated, the Schrödinger equation derives from the wave equation for the wave function ψ, if external potentials are included. It was established, that its formulation in the system at rest – conveniently chosen to be the system of the hydrogen nucleus – requires a time dependent form $\psi(\vec{r}, t)$. The same conclusion can be drawn from the time dependent Schrödinger equation, since the development of ψ in the standard model involves the transformation $\psi(\vec{r}, t) = \psi(\vec{r})e^{-i\omega t}$. In this case the fully expanded wave function of the hydrogen electron for the lowest states will be:

$$\psi_{100}(\vec{r}, t) = C_{100}e^{-r/a}e^{-i\omega_1 t}$$
$$\psi_{200}(\vec{r}, t) = C_{200}\left(1 - \frac{r}{2a}\right)e^{-r/2a}e^{-i\omega_2 t} \qquad (9)$$
$$\psi_{210}(\vec{r}, t) = C_{210}\frac{r}{2a}e^{-r/2a}\cos\vartheta e^{-i\omega_2 t}$$

To calculate the states of electron motion, deriving from the standard solution of the Schrödinger equation, we may employ the wave function Eq. 9 and deduce the corresponding frequencies ω for an arbitrary radius r within the hydrogen shell. For the state (100) we get from:

$$\left(-\frac{\hbar^2}{r^2 2m}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} - \frac{e^2}{r}\right)e^{-r/a} = \hbar\omega(r)e^{-r/a} \qquad (10)$$

the frequency $\omega(r)$:

$$\hbar\omega(r) = \frac{1}{r}\left(\frac{2\hbar^2}{2ma} - e^2\right) - \frac{\hbar^2}{2ma^2} \qquad (11)$$

As could be exspected from the formulation of Schrödinger's equation, the frequencies depend on the distance from the nucleus. Setting $\omega(r)$ to zero and calculating the corresponding radius, the result will be:

$$r(\omega = 0) = (2 - 2)a = 0 \qquad (12)$$

The consequence of the conventional treatment will be *negative* frequencies and *virtual* velocities of the electron wave. Employing a mechanical analogy by asssuming that the electrostatic potential only describes modifications of the electron wave, while kinetic properties are described by:

$$\hbar\omega(r) = \frac{1}{r}\left(\frac{\hbar^2}{ma}\right) - \frac{\hbar^2}{2ma^2} \qquad (13)$$

does not significantly change the result: in this case the radius, where frequencies become negative and velocities consequently virtual begins at $2a$. The result points to a difficulty deterministic theories are generally confronted with: since the energy of individual waves is

generally positive, the reproduction of negative eigenvalues for any specific wave is not possible. This feature usually requires to introduce negative energies by mechanical analogies. But in this case a double perspective – from the viewpoint of intrinsic field and wave properties as well as external mechanical properties – is inherent to every further development of the model, and this double perspective commonly results in major inconsistencies. The same applies to the new framework of material wave theory. If the properties of electrons in the hydrogen system had to be treated on the basis of electrostatic interactions, then a logically consistent model could not be achieved. That electrostatic interactions have been referred to photon exchange and consequently intrinsic field properties of single particles, provides the decisive difference to conventional models.

The calculation of frequencies and physical properties of hydrogen electrons can be repeated for the states (200) and (210) with similar results. It can therefore be concluded, that the standard ground state solutions of quantum theory for the hydrogen atom do not allow for an interpretation of the wave function as a *physical* wave. In the context of the framework developed, the result can be interpreted in two ways: (i) The standard solutions of the Schrödinger equation for the hydrogen atom provides the correct eigenvalues, but the related wave functions are not valid solutions if seen from a physical point of view. (ii) The frequency of the electron within the hydrogen atom is not a valid physical variable.

The first conclusion requires, that the model of hydrogen atoms be reformulated in such a way, that the physical properties of electron waves are accounted for. The second conclusion is only legitimate, if the Copenhagen interpretation [11] of quantum theory is accepted: but as the framework of material waves could already establish, the Copenhagen interpretation has to be rejected, because the wave function of a single particle also possesses physical significance [2].

C. Theoretical limitations

The standard model of hydrogen atoms has been remarkably sucessful in formalizing the experimental results of a wide range of measurements. The concept of electron orbits, structured by radial and rotational states of the hydrogen electron in combination with the intrinsic variable of electron spin allows to account for all effects related to the lines of spectral emission as well as their fine–structure.

The wealth of theoretical and experimental verifications of the standard theory seems to yield any modification either fruitless, since it would be contradicting experimental evidence, or outright senseless, as the theoretical framework is complete anyway. In view of the results of the preceding section, that the eigenstates of electrons due to the standard theory are not valid solutions for *physical* waves, the theoretical development of a framework of physical waves must either stop short of atomic properties, which limits the range of application to electron–photon interactions and the description of single particles, or be extended to account also for atomic properties.

It must be mentioned that a theory of physical waves of the hydrogen electron is severely limited compared to the standard theory, because every assumption must be backed by physical plausibility. While quantum theory can easily postulate the existence of additional parameters, a theory of physical waves *must* at every step be in accordance with (i) the principle of causality, (ii) the basic qualities of particles and waves defined by the extended framework of electrodynamics, and (iii) the modes of interactions defined by photon emission and absorption processes. The result of a theoretical development in this case can be nothing short of a deterministic theory of hydrogen atoms.

Apart from a limitation, these constraints of development are also a theoretical challenge, and it cannot be guaranteed, that the first attempt in this field will provide the ultimate solution to the problem. The current paper therefore proposes what is to be a *dynamic theory of hydrogen atoms*, including the basic experimental results of spectroscopy as well as a limited amount of nuclear physics.

D. Experimental evidence

Due to its importance for the development of atomic models hydrogen is by far the best researched atomic system. Experimental data relevant for a first dynamic model of hydrogen include the following: (i) Thermodynamic data including density at varying thermodynamic conditions [12]. (ii) Spectroscopic data determining the electromagnetic emssion frequency and the structure of emission lines, and x–ray spectroscopy for the measurement of ionization energies [6,13–15]. (iii) And finally, nuclear scattering experiments to determine the dimensions of the nuclear radius [16,17].

III. STRUCTURE OF THE HYDROGEN SHELL

As the standard solutions provided in quantum theory do not account for physical properties of electron waves, the theoretical model of hydrogen atoms has to be reformulated. Contrary to the standard model, the electron in material wave theory does not have to be point–

like, it can be distributed over a finite region of space. This result suggests to give up the distinction between the *electron* and the region of its *probability*–distribution, the shell of the hydrogen atom. Giving up the distinction means, that the electron itself *is* the hydrogen shell, while the internal structure of the shell is determined by internal properties of the *moving* electron. From a theoretical point of view, the internal structure of electrons in motion is described by the wave equation for the wave function $\psi(\vec{r}, t)$:

$$\triangle\psi(\vec{r},t) - \frac{1}{u_{el}^2}\frac{\partial^2}{\partial t^2}\psi(\vec{r},t) = 0 \tag{14}$$

For the electron wave within the basically spherical region of the hydrogen shell, the geometrical features derive from a definite atomic radius and assumed spherical symmetry. This consideration excludes states of motion based on orbital rotations, because (i) they would result in unsteady states along the axis of rotation, and (ii) rotation of electron waves is not limited to discrete frequencies, as can be derived from the modified de Broglie relation for electron waves [2]:

$$\psi(\vec{r},t) = \psi_0 \sin(\vec{k}\vec{r} - \omega t) \quad \vec{k}\vec{r} = \frac{m}{\hbar}(\vec{\omega} \times \vec{r}) \cdot \vec{r} = 0$$
$$\psi = \psi(t) = \psi_0 \sin(\omega t) \tag{15}$$

If motion cannot be lateral motion, then it has to derive from radial oscillations, and the limiting spheres at $r = R_N$, the radius of the proton, and $r = R_0$, the radius of the hydrogen shell must then be functional nodes.

A. Resonant states of motion

These considerations suggest to solve the wave equation for radial symmetry of the wave function ψ, determining the allowed states of electron motion by boundary conditions at the atomic radius R_0. The solutions for $\psi(\vec{r}, t)$ then must be used to determine kinetic properties of the moving electron and the change of electron states finally has to be referred to the observed spectral emissions.

Separating the wave function $\psi = \psi(\vec{r}, t)$ of electron waves into a function $\chi_R(\vec{r})$ and $\chi_t(t)$, where $\chi_t(t)$ shall be periodic with ν_0, the wave equation 14 yields:

$$\triangle\chi_R(\vec{r}) + \frac{\omega_0^2}{u_{el}^2}\chi_R(\vec{r}) = 0 \tag{16}$$

The significance of ν_0, the *resonance frequency* of the hydrogen system, will be shown later on. It derives, essentially, from photon interactions of the nucleus and the electron shell.

For a function $\chi_R(\vec{r}) = \chi_R(r) \cdot Y(\vartheta, \phi)$ with $Y(\vartheta, \phi) = 1$, solutions complying with the boundary conditions at R_0 are, in the simplest case, spherical Bessel functions described by:

$$\chi_R(r) = \frac{\sin(k_n r)}{r} \tag{17}$$
$$k_n = \frac{\omega_0}{u_n} \qquad u_n = \frac{\nu_0 R_0}{n}$$

Density $\rho(r)$ is proportional to $\chi_R^2(r)$, its amplitude can be inferred from electron mass m_e within the shell:

$$\rho(r) = \rho_{el}^0 \frac{\sin^2(k_n r)}{r^2} \qquad \int_{V_s} d^3r' \, \rho(r') = m_e \tag{18}$$

Radial velocities of the electron cannot be generally constant, if the hydrogen atom is stable. They must therefore contain time dependent oscillations, determined by the resonance frequency of the system. Generalizing radial velocity u_n within the shell to a function periodic with ν_0, we may write:

$$u_n(t) := u_n \cos(\omega_0 t) \tag{19}$$

And the radial momentum $p_r(r, t)$ for an arbitrary radius r and an arbitrary moment t is then given by:

$$p_n(r,t) := \rho(r)u_n(t) = \rho_{el}^0 u_n \frac{\sin^2(k_n r)}{r^2}\cos(\omega_0 t) \tag{20}$$

Provided, the constraints are fullfilled, the kinetic state of internal variables is determined at every given moment. The model of hydrogen atoms is therefore deterministic, which is a consequence of internal wave properties, already established to exist beyond the level of uncertainty defined by Heisenberg's relations [10].

B. Electron wave properties

The kinetic properties of the radial electron wave will now be developed for the moment $t = 0$, which will account for the features of the electron wave without electron–proton interations. Including the effects of interactions will be the subject of the following section, since the interplay between the locally periodic structure $\sin^2(k_n r)$ and the oscillation with constant frequency $\cos(\omega_0 t)$ has to be analyzed separately. It will be seen, that the local distribution accounts for the energy levels of the electrons, while constant oscillation results from electron–proton interactions,

The radius of the radial wave nodes are at $r = R_0$, wavelength λ_n increases with increasing n: the wavelength of the electron is therefore higher than the atomic

radius. Using de Broglie's relation for the radial velocity u_n, the wavelength will be:

$$\lambda_n = \frac{h}{m_e u_n} = \frac{h}{m_e \nu_0 R_0} n \tag{21}$$

Applying the dispersion relation $\lambda_n \nu_n = u_n$ the frequency of the radial electron wave will be:

$$\lambda_n \nu_n = u_n \quad \Rightarrow \quad \nu_n = \frac{m_e}{h} \frac{(\nu_0 R_0)^2}{n^2} \tag{22}$$

The kinetic energy of electron motion at $t = 0$ will be, with Eq. 20 and 18:

$$\phi_K(r, t = 0) = \frac{1}{2} \rho_{el}^0 u_n^2 \sin^2(k_n r)$$

$$W_K(t = 0) = \frac{1}{2} u_n^2 \underbrace{\int_{V_S} d^3 r \, \rho_{el}^0 \frac{\sin^2(k_n r)}{r^2}}_{= \, m_e} = \frac{1}{2} m_e u_n^2 \tag{23}$$

The kinetic component covers only half of the total energy, as could be established by the framework of internal particle structures. The additional component, deriving from the electromagnetic properties of particles [2], and total energy of the electron wave will therefore be:

$$W_E(t = 0) = W_K(t = 0) = \frac{1}{2} m_e u_n^2$$

$$W_T(t = 0) = W_E + W_K = m_e u_n^2 \tag{24}$$

$$W_T(t = 0) = m_e \frac{(\nu_0 R_0)^2}{n^2} = h \nu_n$$

The level of electron energy in the hydrogen system is proportional to n^{-2}, which is equivalent to the eigenvalues found by the standard model derived from Schrödinger's equation (see Eq. 7), although, so far, all energy values are positive quantities.

C. Electron–proton interactions

The model developed so far could neither account for binding energy of the electron in the hydrogen system, nor is it a physically valid solution of the problem. The first is a consequence of positive energy values, the latter derives from the fact, that the solution proposed in Eq. 20 does not comply with the wave equation.

$$\left(\triangle - \frac{1}{u_n^2} \frac{\partial^2}{\partial t^2} \right) p_n(r, t) =: \Omega(r, t) \neq 0 \tag{25}$$

Physically, the result signifies, that contrary to motion of a free electron – described by the wave equation – radial electron waves in the hydrogen system are not self–sustained. The electron wave can only prevail due to some kind of interaction accounting for the variation of intrinsic potentials $\phi_E(r, t)$ and $\phi_K(r, t)$. Since the hydrogen system consists only of a nucleus – the proton – and the hydrogen shell – the electron –, interactions must be electron–proton interactions.

In the standard model the interactions are electrostatic interactions, and the potential of interaction is given by Coulomb attraction. By analyzing electrostatic interactions from the viewpoint of internal particle properties, it was found that electrostatic interactions can be referred to an exchange of photons, and that photon emission and absorption processes signify a change of total intrinsic potentials [2]. It could equally be derived, that for two interacting particles the time derivative of emitted and absorbed photon potentials equals zero. With the total potential of electron motion at an arbitrary radius r for an arbitrary moment t:

$$\phi_T(r, t) = \rho_{el}^0 \frac{u_n^2(t)}{r^2} = \rho_{el}^0 \frac{u_n^2}{r^2} \cos^2(\omega_0 t) \tag{26}$$

the time derivative of ϕ_T yields the changes of intrinsic potentials due to non–uniform motion. These changes must lead to photon emission or absorption. If, consequently, the electron potential increases, the reason must be an absorption of photons emitted by the hydrogen nucleus. The same line of reasoning applies to a decrease of electron potential: in this case the electron will be the source of photon emission. The photon potential due to nuclear interactions can then be determined. At r, t the photon potential $\phi_{ph}(r, t)$ of nuclear emission is described by:

$$\frac{\partial \phi_T(r, t)}{\partial t} = -\rho_{el}^0 \frac{u_n^2}{r^2} 2\omega_0 \sin(\omega_0 t) \cos(\omega_0 t)$$

$$\frac{\partial}{\partial t} (\phi_T(r, t) + \phi_{ph}(r, t)) = 0 \tag{27}$$

$$\phi_{ph}(r, t) = \rho_{el}^0 \frac{u_n^2}{r^2} \sin^2(\omega_0 t)$$

Due to the spherical setup and accounting for constant energy flow through sherical surfaces, the total potential of the exchanged photon will be, at the nuclear radius R_N and at the moment $t_{ret} = t \pm \frac{r - R_N}{c}$:

$$\phi_{ph}(R_N, t_{ret}) = \rho_{el}^0 \frac{u_n^2}{R_N^2} \sin^2(\omega_0 t_{ret}) \tag{28}$$

The emitted photon must originate from non–uniform motion of the hydrogen nucleus, using the same procedure, motion at the nuclear surface can then be described by:

$$\frac{\partial}{\partial t} (\phi_{nuc}(R_N, t_{ret}) + \phi_{ph}(R_N, t_{ret})) = 0$$

$$\phi_{nuc}(R_N, t_{ret}) = \rho_{el}^0 \frac{u_n^2}{R_N^2} \cos^2(\omega_0 t_{ret}) \tag{29}$$

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The hydrogen system, in this case, must be seen as a system of coupled oscillations, motion of the hydrogen shell is radial and non–uniform, while interactions are essentially dynamic processes accomplished by photon exchange between the hydrogen nucleus and the electron shell.

D. Resonance frequency ω_0

The properties of nuclear motion have not yet been determined. It can be, initially, concluded, that nuclear potentials at R_N must be related to non–uniform motion of the nuclear surface. This point has, since the first version of the model, been clarified. It was found that oscillation of charge density of the proton is a sufficient condition for the occurence of oscillating fields. These fields are transmitted via photons. The only difference, in the new perspective, is that the whole proton is subjset to density oscillations, not only its surface. See the publication on dynamic charge [18]. The potential of motion must basically comply with the form:

$$\phi_{nuc}(R_N, t) := \rho^0_{nuc}(R_N)u^2_{nuc}(t) \qquad (30)$$

the result suggests to refer the potential to oscillations of the nucleus with a frequency of oscillation equal to ω_0. In this case ω_0, the resonance frequency of the system, derives from material properties of the nucleus, while individual states of radial motion define the state of excitation of the hydrogen system. From Eq. 29 and 30 we get:

$$u^2_{nuc}(t) = u^2_n \frac{\rho^0_{el}}{\rho^0_{nuc}R^2_N} \cos^2(\omega_0 t)$$

$$u_{nuc}(t) = \pm u_n \sqrt{\frac{\rho_{el}(R_N)}{\rho_{nuc}(R_N)}} \cos(\omega_0 t) \qquad (31)$$

$$u_{nuc}(t) = \pm \alpha u_n \cos(\omega_0 t) \qquad \alpha = \sqrt{\frac{\rho_{el}(R_N)}{\rho_{nuc}(R_N)}}$$

Oscillation of the nuclear surface complies with the harmonic equation, as can easily be established:

$$r_N(t) = r^0_N \pm \frac{\alpha u_n}{\omega_0} \sin(\omega_0 t)$$
$$\ddot{r}_N(t)\rho_{nuc}(R_N) = -\kappa \cdot (r_N(t) - r^0_N) \qquad (32)$$
$$\kappa = \rho_{nuc}(R_N)\omega^2_0$$

E. Energy values and atomic radius

The hydrogen system can therefore be interpreted as a *dynamic* system of radial electron waves and coupled electron–proton oscillations, where coupling is effected by photon emission and absorption processes. The energy values describing a specific state of excitation n are therefore: (i) the energy of radial motion of the electron, generally a positive value, and (ii) the energy contained in electron–proton interactions responsible for the energy level of coupling. To estimate the binding energy of an electron in a state n of excitation, it has to be considered, that the undisturbed motion of the electron would signify a total energy of motion equal to $m_e u^2_n$. Due to the interaction process, the average energy in one full period is reduced by:

$$W^0_{el} = m_e u^2_n$$
$$\langle W_{el} \rangle = W^0_{el} \frac{1}{\tau} \int_0^\tau \cos^2(\omega_0 t)dt = \frac{1}{2}m_e u^2_n \qquad (33)$$
$$W_{int} = -\left(W^0_{el} - \frac{1}{2}m_e u^2_n\right) = -\frac{1}{2}m_e u^2_n$$

Contrary to the standard model, the interaction energy of the electron is not independent from its state of excitation. For low excitation values, corresponding to low energy levels of the hydrogen system and $n \to \infty$, the binding energy of the electron aproaches zero. The physical significance of this result will be analyzed later on, currently the important consequence will be, that the physical concept of *ionization* has to be defined differently for the dynamic model of atoms than is usual in quantum theory. For a specific level of excitation n the total energy of the electron and the energy of electron proton interactions are with Eq. 17:

$$W_{el}(n) = m_e(\nu_0 R_0)^2 \frac{1}{2n^2}$$
$$W_{int}(n) = -m_e(\nu_0 R_0)^2 \frac{1}{2\,n^2} \qquad (34)$$

For the calculation of the parameters ν_0 and R_0 the deduced results have to be related to experimental data. We define now the *ionization* energy of the hydrogen system as the binding energy in its highest state of excitation $n = 1$, its value will be about $E_0 = -13.598$ eV. In this case we get:

$$\nu_0 R_0 = \sqrt{\frac{-2E_0}{m_e}} = 2.187 \times 10^6 [ms^{-1}] \qquad (35)$$

The energy value is the energy difference between the hydrogen ground state ($n \to \infty$) and the highest level of excitation and also equal to the energy contained in electron–proton interactions. Ionization energy therefore can either be seen as the quantity of energy necessary to break the bonds of a hydrogen electron or as the quantity of energy necessary to excite the highest state of oscillation.

And the radius of the hydrogen atom can be calculated by employing the relation for the wavelength of radial electron waves Eq. 21, which shall be, for $n = 1$, equal to the atomic radius:

$$R_0 = \frac{h}{\sqrt{2 E_0 m_e}} = 3.33 \times 10^{-10} [m]$$
$$\nu_0 = 6.57 \times 10^{15} [Hz] \tag{36}$$

Comparing with the density of hydrogen in its gas state [12], the total volume of 5.37×10^{22} H atoms (we assume atomic hydrogen, the number of atoms equals the mass of 0.0899 g) comes to 8.38×10^{-3} l, or about 1 % of the total volume of hydrogen gas in standard conditions. In its liquid and solid state the calculated volume of the experimentally measured mass (70.6 – 70.8 g) at low temperatures will be 6.59 l, or about six to seven times the total volume of hydrogen at the conditions of measurements. Accounting for molecular densities by a reduction of about 2 still leads to a volume higher than 3 l. The result indicates that liquid and solid states do not leave the structure of hydrogen atoms or molecules unaffected.

F. Internal fields

The radial and intrinsic momentum of the electron wave and the complementary electromagnetic potential in the hydrogen shell are:

$$\vec{p}(r,t) = \rho_{el}^0 u_n \frac{\sin^2(k_n r)}{r^2} \cos(\omega_0 t)\, \vec{e}^r$$
$$\phi_{int}(r,t) = \rho_{el}^0 u_n \frac{\cos^2(k_n r)}{r^2} \cos^2(\omega_0 t) \tag{37}$$

Using the definition of electric fields [2]:

$$\bar{\sigma}\vec{E} = -\nabla\phi_{int} + \frac{\partial\vec{p}}{\partial t} \tag{38}$$

the change of internal properties can be referred to an external potential $-\nabla\phi_{ext} = \bar{\sigma}\vec{E}$, which must be described by:

$$\nabla\phi_{ext}(r,t) = \nabla\phi_{int}(r,t) - \frac{\partial\vec{p}}{\partial t} \tag{39}$$

Since the setup is spherically symmetric, an integration over a sphere with radius r yields, with the help of Gauss' theorem:

$$\phi_{ext}(r,t) = \phi_{int}(r,t) + \rho_{el}^0 u_n \omega_0 \frac{\sin(\omega_0 t)}{2r} \tag{40}$$

The amplitude of density can be calculated from Eq. 18, together with the expression for velocity u_n from Eq. 17, the potential will be:

$$\phi_{ext}(r,t) = \phi_{int}(r,t) + \frac{m_e \nu_0^2}{2n} \frac{\sin(\omega_0 t)}{r} \tag{41}$$

The first term is the consequence of electron motion, it is the *self interaction term* of the field within the hydrogen shell. The second term must be referred to exterior origins, to the hydrogen nucleus.

The result can be used to determine the nuclear potential of the proton. Since the external potential must result from proton energy, and since the interaction by photons already established harmonic oscillation of the nuclear surface, the periodic features of the potential suggests a relation with the harmonic qualities of the nuclear radius. But if the potential is due to changes of the nuclear radius, the nuclear potential can be calculated.

G. Nuclear potential

The nuclear potential due to a radius $r_N(t)$ may be written:

$$\phi(r_N(t)) := \frac{\phi_N^0}{r_N(t)} \tag{42}$$

And in first approximation the change of nuclear radius will consequently result in periodic fields described by:

$$\phi(r_N(t)) = \frac{\phi_N^0}{r_N^0 \pm \frac{\alpha u_n}{\omega_0}\sin(\omega_0 t)}$$
$$\triangle\phi(r_N(t)) \approx \mp \frac{\phi_N^0}{(r_N^0)^2}\frac{\alpha u_n}{\omega_0}\sin(\omega_0 t) \tag{43}$$

Comparing with the result for the external field in the hydrogen shell and accounting for radial symmetry of the problem will yield, for the intensity ϕ_N^0 of the nuclear field, the expression (nuclear density assumed constant):

$$\frac{(\phi_N^0)^2}{R_N^3} = \text{const} = \nu_0^2 \frac{3\pi^2 m_e m_p}{R_0}$$
$$\frac{(\phi_N^0)^2}{R_N^3} = 5.78 \times 10^{-15} [kg^2/ms^2] \tag{44}$$

The analysis of proton potentials can be carried one step further, touching the fundamental problem of proton stability. As the framework of nuclear forces in the current standard is completely separated from the interactions in atomic or macro physics, it cannot easily be compared to the nuclear potentials derived. The dynamic theory of hydrogen atoms, furthermore, is limited to only two material parameters, density of mass and density of charge. These limitations would either require an extension of particle properties in the hydrogen nucleus – which is, essentially, what the current standard of QCD (quantum chromo dynamics) is based on – or a modification of existing parameters to account for the extreme

properties of nuclear mass. Estimating the nuclear potential due to gravity or electrostatic interactions, we get for standard coupling and constant nuclear density with:

$$\nabla \vec{G} = -\gamma \rho_N \quad \nabla \vec{E} = +\frac{1}{\epsilon}\sigma \tag{45}$$

for the nuclear potential the result (*grav* refers to gravitational, *el* to electrostatic origins):

$$\phi_{grav} = -\gamma \frac{m_p \rho_N R_0 \alpha}{8\pi^2 r_N} \frac{\sin(\omega_0 t)}{nr}$$
$$\phi_{el} = +\frac{e\sigma_N R_0 \alpha}{32\pi^3 \epsilon r_N} \frac{\sin(\omega_0 t)}{nr} \tag{46}$$

And since this potential is the origin of forces on the electron wave given by Eq. 40, the required nuclear radius will be described by:

$$\epsilon\, r_N^{7/2} = \frac{6e^2}{32\pi^3 m_e \nu_0^2}\sqrt{\frac{2m_e R_0}{3m_p}} = 1.38 \times 10^{-48}$$

$$\gamma^{-1}\, r_N^{7/2} = \frac{6m_p^2}{32\pi^3 m_e \nu_0^2}\sqrt{\frac{2m_e R_0}{3m_p}} = 4.70 \times 10^{-63} \tag{47}$$

From the known coupling constants the nuclear radius can be calculated. It would be for electrostatic or gravitational coupling:

$$r_N(grav) = 3.69 \times 10^{-21} m$$
$$r_N(el) = 3.05 \times 10^{-11} m \tag{48}$$

The nuclear radius, in case of gravity couplings, would be far too small, in case of electrostatic coupling far too big to be consistent with nuclear scattering experiments. It can therefore be excluded, that nuclear stability can be the result of any usual field we deal with in quantum theory or electrodynamics. The same conclusion has been drawn in the standard theory, and the consequence was the development of theories on nuclear forces as well as the extended framework of QCD.

H. General solutions

The solution for radial motion of the electron wave in the hydrogen atom applied spherical symmetry. In a more general picture, this form of motion of the electron wave and consequently of the nuclear surface will only be the simplest form of motion, the monopole oscillation of the nuclear surface. In a more general picture the nuclear surface is an oscillating multipole, which must affect the electron wave in such a way, that the lateral components of motion, the spherical harmonics $Y_{lm}(\vartheta, \varphi)$ are no longer constant. The constraints for a general solution of the problem then have to be accounted for in such a way, that the wave equation for the intrinsic electron momentum $\vec{p}(\vec{r}, t)$ is satisfied due to the interactions with nuclear potentials. The system again has to be a system of coupled oscillations, and the energy levels can be determined by calculating, in the same manner, kinetic energy components and components of interaction.

I. Summary

In this section we solved the wave equation of the hydrogen electron for spherical symmetry. The resulting states of radial motion could be related to the energy levels of excited hydrogen by including the experimental value of ionization energy. The interaction energy of nuclear oscillation and electron motion accounted for the periodic feature of electron oscillation, the binding energy of an electron was found half of the energy contained in its radial motion. Internal fields of electron–proton interaction were referred to nuclear potentials, and by estimating the effects of electromagnetic and gravitational coupling it could be established, that intrinsic nuclear interactions cannot be explained by this type of interaction. We equally showed that the radially symmetric solution is but the simplest case, and that nuclear multipole oscillation provides additional modes of electron–proton interactions as well resonant states of electron motion.

IV. PHOTON EMISSION

A. Balmer's relation

The energy levels of electron motion within the hydrogen shell account for the discrete frequency of spectral emission, if the change of energy from a level n to a level m is emitted as photon energy, since with 33 the energy difference must be:

$$\triangle W_{nm} = \frac{1}{2}m_e(u_n^2 - u_m^2) = \frac{m_e}{2}(\nu_0 R_0)^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \tag{49}$$

And using Planck's relation for the frequency of the emitted photon, the result will be Balmer's relation:

$$\nu_{nm} = \frac{m_e(\nu_0 R_0)^2}{2h}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = \nu_{em}^0\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$
$$\nu_{em}^0 = 3.288 \times 10^{15} Hz \tag{50}$$

The result, although defining the allowed emission frequency of photons, does not provide any insight into the process of emission. Within the framework of a dynamic model of atoms it is therefore only half of the whole story.

While induced emission, the transition due to external origins, can be seen as a process determined by interactions, the same does not apply to spontaneaus emissions, which must be referred to internal origins. If spontaneous emissions occur, then the theoretical framework must provide a model which meets (i) the properties derived for the intrinsic variables, and (ii) the requirements of thermodynamics if a statistical ensemble of atoms is considered.

That spontaneous emissions occur is suggested by the principles of thermodynamics, which require that entropy in natural processes increases, which means, that atoms with a high level of excitation must in a closed system emit their energy until thermodynamic equilibrium, corresponding to the maximum of entropy, is achieved. We account for these requirements by a model, which limits statistical considerations to inter–atomic processes, while the process of emission itself shall be deterministic.

B. Causality and probability

The transition of electron motion from a level n to a level m in the standard theory involves selection rules and probability considerations. The procedure reflects the interpretation of electron waves as probability waves and is a conesequence of the fundamental assumptions in quantum theory, as already demonstrated [2].

In the framework of material wave theory, events and processes on the level of electron waves are described, although only partly, by Maxwell's equations, the events are therefore determined. Which means, that probability cannot enter at this level of theoretical formulations, but must be accounted for on the level of thermodynamic environments. The emission of photons as well as the transition from one state n to a state m is therefore related to probability only insofar it refers to the mechanical and electromagnetic interactions of one atom with other atoms. Excitation and emission then are processes governed by the general rules of Maxwell–Boltzmann distributions, where the probability of a state E_n of atomic excitation can be described by:

$$W(n) \propto e^{-E_n/k_B T} \qquad (51)$$

The probability of higher levels of excitation is generally very low, most hydrogen atoms will be in a state $n > 100$ of excitation. The result suggests, that spectroscopic data based on the frequency range of visible or near visible emisssion, should concern only an insignificant minority of physical processes in a given environment. The probability of individual states n has been plotted in Fig. 1, it can be seen, that the transition interval from probability 1 to probabilities close to 0 in standard conditions encompasses the range of states from $n = 10$ to $n = 100$.
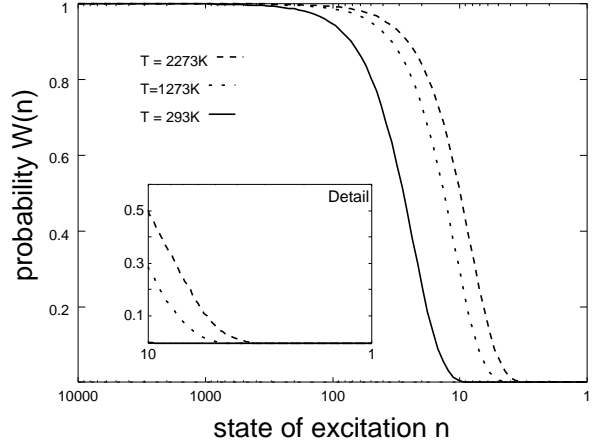

FIG. 1. Probability of states of excitation n at standard conditions 293K, at 1273K and at 2273K. The probability of excited states with $n < 10$ is insignificant compared to states with $n > 100$

That interactions of the atom with its environment are probabilistic does not imply, though, that the transitions internally are also subject to the same consideration. It can equally mean, that the internal processes are strictly determined, once the initial energy value is acquired by inter–atomic processes. We develop the consequence of this concept for the process of photon emission by modifying the relations of electron and nuclear excitation to account for decay processes. Since the hydrogen atom is a coupled system of nuclear and electron oscillations, decay of either electron motion or nuclear oscillation will have the same effect: the level of excitation is reduced.

C. Deterministic model

Interactions between nuclear oscillations and radial electron waves were the origin of internal stability of excited atoms. The problem of spectral emission of non-uniform motion can be treated in a deterministic way by an additional assumption: nuclear oscillations shall not be constant, but subject to decay. Then the transition from resonant state n $\rightarrow m$ can be based on the formulation for nuclear velocity $u_{nuc}^2(t)$:

$$\rho_{nuc}^0 u_{nuc}^2(t) = \rho_{el}^0 u_n^2 e^{-\alpha t} \qquad \alpha = \frac{2}{\tau_\epsilon} \ln \frac{m}{n} \qquad (52)$$

where τ_ϵ is the emission interval. If nuclear motion is altered, the stability of radial motion is no longer sustained, and the time derivative of total potential at any arbitrary point of the shell is not zero:

$$\frac{\partial \phi_T^{nuc}}{\partial t} + \frac{\partial \phi_T^{el}}{\partial t} = \frac{\partial \phi_T}{\partial t} \neq 0 \qquad (53)$$

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We assume now, that the time derivative of total potential is emitted in the form of photons. Photon energy density can then be calculated, and integrating over the atomic shell will allow to determine the frequency of spectral emission. Setting total potentials of nuclear and electron motion equal to:

$$\phi_T^{nuc}(r,t) = \frac{\rho_{el}^0 u_n^2}{r^2} \sin^2(\omega_0 t) \, e^{-\alpha t}$$

$$\phi_T^{el}(r,t) = \frac{\rho_{el}^0 u_{el}^2(t)}{r^2} \cos^2(\omega_0 t) \tag{54}$$

The time derivatives of total potential and consequently the total potential of emission at r yields a non–linear differential equation, which can be transformed into a linear one by substituting $z(t) = u_{el}^2(t)$:

$$z'(t) + z(t)\frac{g(t)}{f(t)} = -\frac{h(t)}{f(t)} \tag{55}$$

$$f(t) = \cos^2(\omega_0 t) \qquad g(t) = 2\omega_0 \cos(\omega_0 t)\sin(\omega_0 t)$$
$$h(t) = u_n^2 e^{-\alpha t} \left(2\omega_0 \cos(\omega_0 t)\sin(\omega_0 t) - \alpha \sin^2(\omega_0 t)\right) +$$
$$+ \frac{r^2}{\rho_{el}^0}\frac{\partial \phi_T^{ph}}{\partial t}$$

In first approximation, assuming the time derivative of photon potential constant, the general solution will be:

$$z(t) = \frac{1}{M(t)}\left\{-\int \frac{h(t)}{f(t)}M(t)dt + C\right\}$$

$$M(t) = \exp\left(\int \frac{g(t)}{f(t)}dt\right) \tag{56}$$

which yields the equation for electron velocity:

$$u(t) = \frac{1}{\cos(\omega_0 t)}\sqrt{C - u_n^2 e^{-\alpha t}\sin^2(\omega_0 t) - \frac{r^2}{\rho_{el}^0}\frac{\partial \phi_T^{ph}}{\partial t}\,t} =$$
$$= \frac{1}{\cos(\omega_0 t)}\sqrt{C - \frac{r^2}{\rho_{el}^0}\left(\phi_T^{nuc}(r,t) - \frac{\partial \phi_T^{ph}}{\partial t}\,t\right)} \tag{57}$$

The constant C can be calculated from the intial value of u (t), while the value for $t = \tau_\epsilon$ allows to determine photon energy:

$$u^2(0) = C = u_n^2$$
$$u^2(\tau_\epsilon) = u_n^2 - \frac{r^2}{\rho_{el}^0}\frac{d\phi_T^{ph}}{dt}\tau_\epsilon = u_m^2$$

$$\frac{\partial \phi_T^{ph}(r,t)}{\partial t}\tau_\epsilon = \phi_T^{ph}(r) = \frac{\rho_{el}^0}{r^2}\left(u_n^2 - u_m^2\right) \tag{58}$$

And substituting the variable $x = t/\tau_\epsilon \quad x \in [0,1]$, the potential of electron motion during emission which, for the sake of simplicity, is taken to prevail for $\tau_0/2$ seconds:

$$\phi_T^{el}(x,r) = \frac{\rho_{el}^0}{r^2}u^2(x)\cos^2(\pi x) =$$
$$= -\phi_T^{nuc}(x,r) + \frac{\rho_{el}^0}{r^2}\left(u_n^2 - \left(u_n^2 - u_m^2\right)\cdot x\right) \tag{59}$$

A simple integration over the electron mass within the atomic shell then yields the frequency and energy of photon emission:

$$E_{nm}^{ph} = h\nu_{nm}^{ph} = \frac{m_e}{2}\left(u_n^2 - u_m^2\right)$$
$$\nu_{nm}^{ph} = \frac{m_e(\nu_0\,R_0)^2}{2h}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \tag{60}$$

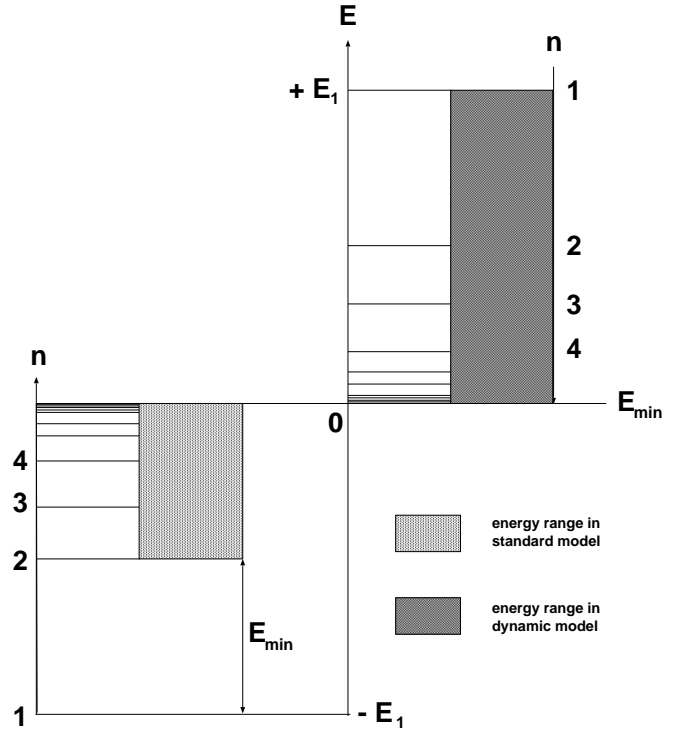which is exactly the result already derived and which is equal to Balmer's law.

FIG. 2. Energy terms and interaction energy due to the principal quantum numbers n in quantum theory (left) and material wave theory (right). Total energy at the level $n = \infty$ in both cases equals zero. Total energy in quantum theory is generally negative, in material wave theory positive. The interaction energy in quantum theory must be higher than $\triangle E = E_2 - E_1$, while in material wave theory it can be arbitrarily low

The actual dynamical reason leading to exponential decay of nuclear oscillation is well beyond the scope of this paper. But if nuclear oscillations are not stable, then there seems to exist no reason, why emission shall not continue in the following interval. If the parameter of nuclear damping α remains constant, then the emission interval will in general not be equal to $(2k+1)\tau_0/2$, but

different for every single emission process. Equally will different decays $n \rightarrow m_1$ and $n \rightarrow m_2$ correspond to different emission intervals:

$$\frac{\tau_{\epsilon_1}}{\tau_{\epsilon_2}} = \frac{\ln m_1 - \ln n}{\ln m_2 - \ln n} \qquad (61)$$

Excited atoms will thus lower their level of excitation by continuous emissions until they are again excited. The process of excitation and emission - whereby excitation does not seem as easily accessible to formalisation - is then a basically statistical problem, and while the emission process is purely deterministic, the interactions with the environment are not.

D. Thermodynamic environment

The essential difference between the standard model of hydrogen atoms and the current one concerns the range of allowed energies. While hydrogen atoms in quantum theory generally have a negative level of total energy, the energy in the current model is generally positive. And while the hydrogen ground state in quantum theory is described by $n = 1$, the ground state in the current model is given by $n = \infty$, which describes the level of absolute inertia, the point where $T = 0K$ (see Fig. 2).

Therefore every level of atomic excitation up to the level $n = 1$ allows for electromagnetic emissions, and hydrogen emissions are bound to occur well below the visible or infrared spectrum. The transition depends, as previously calculated, on the emission interval, which should be affected by the enviroment of an emitting hydrogen atom. The frequency of emission is given by allowed states of radial motion. Statistical considerations, although without any relevance to emission frequencies, therefore enter the picture in the calculation of emission intensities of a specific line of the hydrogen spectrum.

V. CONCLUSION

We have analyzed, in this paper, whether the standard solution of the hydrogen atom, given in quantum theory, is compatible with the dynamic and intrinsic properties of electrons. From this analysis it was concluded that the standard model is essentially invalid, since it does not present a physical (as opposed to an only *algorithmic*) solution to the problem of hydrogen spectra. An alternative and physical model of electron waves within the hydrogen atom was developed, it was shown compatible with the experimental results of hydrogen spectra. The model reversed the standard scaling of the electron energies, since the state of $n = 1$, which is identified as the hydrogen ground state in quantum theory, is the state

of highest excitation in microdyanmics. The energy of a hydrogen atom depends on the interactions with the environment and is generally positive. A deterministic and local picture of the emission process could be deduced, which is based on the decay of nuclear oscillations. It could also be shown that the nuclear potential cannot be of electromagnetic or gravitational origin, which seems to necessitate a separate and different mode for the interactions in a nuclear environment.

VI. DISCUSSION

The model of hydrogen, developed in this paper, reveals three interesting differences to the standard model:

- The radius is not changed if the atom is excited, contrary to the standard conception of a Rhydberg atom.

- The liquid and solid state is not merely a combination of two identical atoms, but affects the structure and hence the physical qualities of single atoms decisively.

- There is no minimum excitation energy: although the next higher state of excitation in a given environment requires a discrete amount of energy, this amount is not independent of the thermodynamic environment.

These changes affect the concept of hydrogen atoms substantially. Instead of well-defined and seemingly persistent structures the dynamic atoms not only change their electron-proton interaction energy with their state of excitation, but also their very structure in any liquid or solid aggregation with other atoms. From a rather philosophical point of view this indicates a departure from the very notion of the atom (in its original conception an undividable and unchanging entity). The validity of this new model has to be subjected to experimental tests before it can become part of physical reality, and the measurements to be performed are currently developed. Apart from this question the possibility to formulate a dynamic and hence causal model of hydrogen atoms - whether this first model will ultimately be suitable or not - indicates, in a way, a major breakthrough from the purely statistical description of quantum mechanics.

From the viewpoint of thermodynamics the model seems, on first glance, better suited to describe the third principle. The energy differences possess a vanishing low energy limit, whereas the conventional atom in quantum theory is characterized by vanishing high energy limits. How this feature translates into experimental results has to be worked out. But it could, eventually, provide a

test to differ between the results of microdynamics and quantum mechanics.

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